Putting Fractions in Their Place

Leslie Blackwell Galen

1. INTRODUCTION. Mathematicians need fractions, certainly more than fractions need mathematicians. Any respectable book on mathematics, from an elementary primer to the most esoteric monograph, is loaded with fractions carrying out their manual labor of division. But as all these books show plainly, it is not enough to merely think a fraction; the fraction must be enunciated—that is, put on paper—for it to be of any use.

Here we will try to make it easier to understand the fraction, not as a mathematical word, so to speak, but as a typographical entity with many faces and many means of expression. In doing so we hope to help a few authors, whose manuscripts come back bleeding from the copyeditor; a few students, who wonder fleetingly why the same (is it the same?) fraction looks so different from place to place; and a few editors, who gamely fight the good fight to achieve four goals. The first is clarity, that is, clear communication from author to reader. The second is legibility, which entails setting the type so that the myopic reader can read without squinting. The third goal is consistency: every category of fraction should set the same way throughout. The fourth goal is conservation of space.

Though not as important as the first three goals, this last item bears examining. We must swallow our pride and admit that, although worthy, useful, and fascinating, mathematics (especially higher mathematics) is not very popular with the masses. No calculus textbook is likely ever to make the *New York Times* best-seller list, nor will this MONTHLY sell as many copies as *People*. The conclusion we must reach, then, is that mathematics is not terribly *profitable*, either for those who engage in it or for those who publish it. In order to publish as much as can be done under the circumstances, publishers use many methods to *conserve*: paper, labor, keystrokes, money. The proper use of fractions can be one small weapon in the battle to keep costs down, while not compromising quality.

2. TYPES OF FRACTIONS. Fractions are set in four distinct ways. Each has its beauty and its utility, and each is prone to abuse. For now we'll show all fractions as *displayed*, that is, by themselves with no text around them. Notice how much vertical space we must use to do this.

A *case fraction* is a small, upright fraction:

$$\tfrac{1}{2}$$

These fractions are most useful in running text, where they take up no extra space and allow the author's thought to flow more freely.

> Since 1, -1, $\tfrac{1}{2}$, and 2 are zeros of the polynomial, the Factor Theorem implies that the linear expressions $x-1$, $x+1$, $x-\tfrac{1}{2}$, and $x-2$ are factors of the polynomial.

This tiny fraction should be used only for what we in the publishing industry call "simple" or "common" fractions. Exactly what this means depends on whom one asks.

The following is my own rule for what can be set as a case fraction: small arabic numbers, preferably no more than two in either numerator or denominator. That's it. Greek letters, operators, super- or subscripts, even parentheses, must be avoided when using a case fraction, for the obvious reason that a number set as a case fraction (given the standard 10-point type size of a mathematics book) is only 7-point type. Trying to read anything complex in 7-point type is difficult at best. At worst, this tiny splotch of black ink will be misread or even skipped over by the tired eyes of the reader: $\frac{1}{4}$ is fine; $\frac{49+144-81}{2(84)}$ is not.

The second kind of fraction is called a *special fraction*, and it is. These are set small, like the case fraction, but with a slanted solidus:

½

Sadly, this fraction is never used in modern mathematical notation. The special fraction resides comfortably in the home of mathematics's domestic cousin, the cookbook, where "add 1½ cups sugar" is the fraction's invitation to us to measure and taste.

Third is the *shilling fraction*, a recumbent fellow. The type here sets full size, but the solidus sets slanted:

$$1/2$$

This fraction sets splendidly in text, makes superscripts and subscripts easy to read, and removes confusion from the dreaded fraction-within-a-fraction:

$$\frac{(x-2)^2}{4/9} = 1$$

If this way of setting is unbearable, try setting it as case:

$$\frac{(x-2)^2}{\frac{4}{9}} = 1$$

But setting it built-up, a notion that we discuss shortly, makes it hard to read and takes up too much space:

$$\frac{(x-2)^2}{\dfrac{4}{9}} = 1$$

By using shilling fractions, a larger, more complicated, more subtle expression can nestle in a sentence. Notation that should not be present in a case fraction can reside easily here:

> The graph of $y = f(x)/g(x)$ crosses the x-axis when both $f(x) = 0$ and $g(x) \neq 0$.

Exposition is clear, and precious vertical space is saved.

Three cautions concerning shilling fractions: First, a *very* long and complex fraction set "shil" can be somewhat hard to follow, especially on first reading, e.g., $b = (x_0 - x_2)^2[f(x_1) - f(x_2)] - (x_1 - x_2)^2[f(x_0) - f(x_2)]/(x_0 - x_2)(x_1 - x_2)(x_0 - x_1)$. Next, we must be careful in using surrounding parentheses, so that the reader does not mistake what is actually a part of the fraction. Thus, without clarifying parentheses, the fraction $1 - x/y$ might read as either

$$1 - \frac{x}{y}$$

or

$$\frac{1 - x}{y}$$

to the inexperienced eye. Finally, chances are good that a long shilling fraction in running text will not fit on one line and must then be broken, with some of the denominator (usually) spilling onto the next line. My own practice in this instance is simply to break the paragraph, set the fraction in display by itself, and keep moving. This idea brings us naturally to the billboard of fractions, the built-up fraction.

Built-up fractions, often denoted b/u by editors, are full size with a horizontal solidus:

$$\frac{b_{i,N+1} - \sum_{j=i+1}^{N} b_{i,j} q_{j-n}}{b_{i,i}}$$

This method of setting fractions allows for maximum impact, with none of the faults of the previous options. The built-up fraction announces its presence with authority. Because of this, one often sees built-up fractions that are quite capable of being set another way, but are set alone for emphasis, sometimes with a corresponding equation number for convenient subsequent reference.

This fraction is the one most liable to abuse, and the most common mistake made with built-up fractions is overuse. Here is an example:

> Using the point-slope equation with the point (2, 3), we have
>
> $$y - 3 = \frac{1}{2}(x - 2).$$
>
> If we use the point-slope equation with the point (–4, 0), we have
>
> $$y - 0 = \frac{1}{2}\big(x - (-4)\big)$$
>
> or
>
> $$y = \frac{1}{2}(x + 4).$$

Despite the many instances where emphasis is more important than space-saving, and despite an author's conviction that each instance of a built-up equation is vital to pedagogy, we must plead with authors to stop the madness. Overuse of the built-up fraction is much like overuse of boldface text, or cologne, or advertising. Overuse dulls the impact. If an author truly believes that a reader will not comprehend a fraction unless it is displayed, then he or she has a poor opinion of the audience for whom mathematics is intended. The example just cited reads every bit as smoothly and comprehensibly in the following form:

Using the point-slope equation with the point (2, 3), we have $y - 3 = \tfrac{1}{2}(x - 2)$. If we use the point-slope equation with the point $(-4, 0)$, we have $y - 0 = \tfrac{1}{2}(x - (-4))$ or $y = \tfrac{1}{2}(x + 4)$.

How much use is overuse? It depends. But everyone has read examples of it.

A more pernicious kind of abuse is putting built-up fractions *within* text. Every mathematics editor whom we know has received manuscripts containing passages that look like this:

Let $f(x) = \dfrac{x + a}{x + b}$, where $a \neq b$. Show that f^{-1} exists and find $f^{-1}(x)$. Give the domain and range of f and f^{-1}. Verify that $(f^{-1} \circ f)(x) = x$ for all x in the domain of f and $(f \circ f^{-1})x = x$ for all x in the domain of f^{-1}.

To set a built-up fraction within text destroys the text's linespacing, or *leading* (measured from baseline to baseline). Some authors argue that this interruption of smooth flow is irrelevant, and that it actually saves space by not setting an item in display. The latter argument may be marginally true, but the former is definitely not. Physically, the eyes travel across a line of text at a comfortable pace, and fall naturally to the continuing line below. Such a large chunk of mathematics, followed by more space between lines than there should be, distracts the reader, who must then take a second to reorient, discover where the eyes stopped, and spend another annoying second to find that continuing next line. It sounds trivial, but it isn't: reading mathematics is hard work, often exhausting work, and we shouldn't make it harder just out of obstinacy. The offending line in the foregoing passage can be fixed easily to read as a shilling fraction: Let $f(x) = (x + a)/(x + b)$, where $a \neq b$.

3. A FEW TIPS AND TRICKS. Don't always set simple fractions as case. For example, simple fractions within a larger, displayed equation that already contains built-up elements (sums, integrals, other built-up fractions) should be built up to match.

$$(\sin x)^4 = \bigl((\sin x)^2\bigr)^2 = \left(\frac{1 - \cos 2x}{2}\right)^2 = \frac{1}{4}\bigl(1 - 2\cos 2x + (\cos 2x)^2\bigr)$$

But there is no need to set simple fractions built up in a displayed equation just because it's displayed:

$$p_2 = \tfrac{3}{20},$$

not

$$p_2 = \frac{3}{20}$$

Other hints on setting fractions:

- Fractions as super- or subscripts should *always* be set as shilling ($\alpha^{x/2}$, never $\alpha^{\frac{x}{2}}$) for readability.
- Any fraction that sets as a numerator or denominator in a larger fraction should set shilling, mainly to avoid confusion over which solidus is which.
- Fractions can appear differently to meet different requirements, even in the same section. A simple fraction might thus appear as case in the text body, shilling in a

superscript, and built-up in a displayed equation. Consistency throughout is the key to ultimate legibility.

- Fractions set within a square root symbol should set shilling in text, but built-up in display. A fraction like $f(x) = \sqrt{(x+2)/(x-1)}$ sets shilling in text, but

$$f(x) = \sqrt{\frac{x+2}{x-1}}$$

in display. But $\sqrt{1/3}$, though a simple fraction, would disrupt the linespacing if it were set in case, so we set it shil here. In display it can set either case or built up:

$$\sqrt{\tfrac{1}{3}}, \quad \sqrt{\frac{1}{3}}$$

- Fractions set within matrices can be set any appropriate way, but should be consistent, both within a matrix and throughout; i.e., set all case, all shilling, or all built-up (if one must), but not bits of each.

These are by no means absolute rules. Mathematics, like any language, can be adjusted and modified to address a specific need. Nor does this author claim any kind of moral or editorial high ground, only the hard experience of reading, cover to cover, and editing more than three hundred mathematics and science books. The hope here is to clarify, not to dictate; to begin a dialogue, not to end one; and to further the cause of clarity, elegance, and beauty that is the final proof of mathematics.

ACKNOWLEDGMENTS. I am grateful to my colleagues Professors J. Douglas Faires and James DeFranza, who graciously allowed me to (ab)use their book *Precalculus* (just released in its third edition) and who provided valuable feedback. I also thank Dianne Parish, who typeset this article, and Don DeLand, who encouraged me to write it.

REFERENCES

1. Ellen Swanson, *Mathematics into Type*, revised edition, American Mathematical Society, Providence, 1986.
2. J. Douglas Faires and J. DeFranza, *Precalculus*, 2nd ed., Brooks/Cole, Pacific Grove, CA, 2000.

LESLIE BLACKWELL GALEN is the Managing Editor at Integre Technical Publishing Co., which composes this MONTHLY. She earned a B.A. in philosophy from the University of Colorado.
Integre Technical Publishing Co., 4015 Carlisle NE, Suite A, Albuquerque, NM 87107
leslie@integretechpub.com